region of kindled slices in the presence of blockers of excitatory amino acid receptors with the aim to release endogenous Zn^{2+} onto the granule cells. In one of three experiments, we successfully reproduced the effects of perfused Zn^{2+} on sIPSCs, presumably through the release of Zn^{2+} from sprouted mossy fibers. Repetitive stimuli delivered to the same location in control slices had no effect on sIPSCs (n = 6). In slices, Zn^{2+} release experiments are difficult to control because even low-frequency stimuli used to test evoked responses can inadvertently release the bulk of Zn^{2+} from the mossy fibers. In the absence of any exogenous Zn^{2+} added to the ACSF, the lost Zn^{2+} cannot be replenished (C. J. Frederickson, personal communication).

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TECHNICAL COMMENTS

Analog Computational Power

Response: Peter Shor (*1*) and Richard Y. Kain (*2*) recently commented on my report "Computation Beyond the Turing Limit" (*3*). Shor questions the nature of the advice used in analog computation, or equivalently, the real weights in the neural networks model. He suggests that they must either be programmed, or be random, or be physical constants, and notes problems in all these cases.

First, by definition, the constants are not necessarily programmable, because they are general, not only computable, real numbers.

Second, the constants are inherently different from random numbers. Rather, they compose the real characteristics of a system. To exemplify this, consider the logistic map

$$x_{n+1} = ax_n(1 - x_n)$$

(*4*) (the state of the system at time n is represented by x_n). Here a minute change in the constant a may result in a qualitative change in the motion, such as period doubling or a transition from periodic to chaotic motion. Independent of this, let me illuminate random processes. Computer scientists often model a random process as a coin that has a success probability (to fall on "heads") of 1/2. Probabilistic Turing machines that use a coin with (exactly) 1/2 success probability compute the class BPP (which, as Shor says, is believed to be no stronger than what deterministic Turing machines compute efficiently). However, I have shown (*5*) that if the success probability of the coin is a real number, the resulting class is again super-Turing! (The networks in this case compute the class BPP/log, which is included in P/poly.) The exact probability of the coin is not known to the Turing machine (nor the underlying process) that utilizes it and can only be approximated by a chain of flips; yet, it still adds power to the classical model.

Third, the weights in neural network models can be thought of as modeling the physical characteristics of a specific system. Shor's comment that the current measurements of physical constants are poor is crucial if one wants to build a general analog computer directly from the description; the design of such a computer is an open problem. However, this problem is immaterial for the mathematical modeling of an analog computation of nature.

In a natural analog computation process, one starts from initial conditions that constitute the (finitely describable) input, and the system evolves according to the specific equations of motion to its final position, which constitutes the output. The evolution is controlled by the exact equations of motion with the exact physical constants. The analog physical system "solves" the equations of motion exactly. For example, planetary motion is used to measure time with very high precision although we know the gravitational constant G only to two digits. The planets, of course, revolve according to the exact value of G, irrespective of its measurement by humans.

Although the networks are defined with unbounded precision, up to the qth step of the computation, only the first $O(q)$ bits in both weights and activation values of the neurons (and the first log q bits that describe the stochastic process) influence the result (*6*). This property of neural networks is identical to that of chaotic systems, suggesting that neural networks are indeed natural models of analog physical dynamics.

In his comment, Kain does not mention the importance of constraints in computation as established by Karp (*7*) and others. The imposition of constraints is one of the main developments that revolutionized the classical theory of computation from discrete mathematics into the modern complexity theory of realistic machines. Readers interested in pursuing some of the issues raised by Kain (for example, the difference between oracle and advice machines, as well as complexity) are referred to (*8*) for their precise description. To summarize, both under resource constraints (complexity) and in their absence (computability), my model exceeds the Turing power, and thus may be referred to as a "super-Turing" one.

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